# Deriving Vacuum Permittivity from Fundamental Dimensional Symmetry in Laursian Dimensionality Theory

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**Abstract**

Following our earlier derivations of the Planck constant and elementary charge, this paper completes the foundational framework of electromagnetic constants by deriving vacuum permittivity ($\varepsilon_0$) from fundamental dimensional symmetry. Within Laursian Dimensionality Theory (LDT), which interprets spacetime as a "2+2" dimensional structure—two rotational spatial dimensions plus two temporal dimensions—vacuum permittivity emerges as a necessary coupling parameter between electromagnetic fields and this dimensional structure. We demonstrate that $\varepsilon_0 = \frac{e^2}{4\pi\hbar c\alpha}$, where $e$ is the elementary charge, $\hbar$ is the reduced Planck constant, $c$ is the speed of light, and $\alpha$ is the fine structure constant. This expression gains profound significance when combined with our previously derived relationship $e = q_P\sqrt{\alpha}$, where $q_P$ is the Planck charge. Together, these relationships reveal vacuum permittivity not as an arbitrary constant, but as a fundamental measure of how electromagnetic fields couple to the rotational dimensions of spacetime. The product $\varepsilon_0\mu_0 = \frac{1}{c^2}$ directly reflects the dimensional coupling factor $\frac{t^2}{d^2}$ that relates temporal dimensions to rotational dimensions. This dimensional approach transforms our understanding of electromagnetic constants from empirical values to geometric necessities arising from the fundamental structure of reality.

## 1 Introduction

Vacuum permittivity ($\varepsilon_0$), along with vacuum permeability ($\mu_0$), constitutes one of the most fundamental parameters in electromagnetic theory. It appears in Coulomb's law, characterizes the electric field energy density in free space, and determines the speed of electromagnetic waves through the relationship $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$. Historically, $\varepsilon_0$ was treated as an empirical constant, and more recently in the SI system, it has been defined indirectly through the fixed values of $c$ and $\mu_0$.

Despite its central role in physics, the deeper physical meaning of vacuum permittivity has remained elusive. Why does empty space have this specific permittivity value? Is it truly a fundamental constant, or does it emerge from a deeper structure of reality?

In previous work, we introduced Laursian Dimensionality Theory (LDT), which proposes a radical reinterpretation of spacetime as a "2+2" dimensional structure: two rotational spatial dimensions plus two temporal dimensions, with one of these temporal dimensions typically perceived as the third spatial dimension. This framework emerged from a reformulation of Einstein's mass-energy equivalence from $E = mc^2$ to $Et^2 = md^2$, where $c$ represents the ratio of distance ($d$) to time ($t$).

Within this framework, we have already derived expressions for the Planck constant as $h = 2\pi E_P t_P$ (where $E_P$ is the Planck energy and $t_P$ is the Planck time) and the elementary charge as $e = q_P\sqrt{\alpha}$ (where $q_P$ is the Planck charge and $\alpha$ is the fine structure constant). Both derivations have demonstrated extraordinary numerical precision, revealing that these fundamental constants emerge from the dimensional structure of spacetime rather than existing as independent parameters.

In this paper, we extend our approach to vacuum permittivity, completing the foundational framework of electromagnetic constants within LDT. We demonstrate that $\varepsilon_0$ is not an arbitrary constant but emerges naturally from the coupling between electromagnetic fields and the rotational dimensional structure of spacetime. This insight not only deepens our understanding of electromagnetic phenomena but also strengthens the case for LDT as a unifying theoretical framework that reveals the geometric origins of fundamental physical constants.

# 2 Theoretical Background

## 2.1 Laursian Dimensionality Theory

Laursian Dimensionality Theory begins with the reformulation of Einstein's energy-mass relation from $E = mc^2$ to $Et^2 = md^2$. This seemingly simple algebraic manipulation reveals a profound insight: the squared terms suggest that space has two rotational dimensions ($d^2$) while time has two dimensions ($t^2$).

This leads to a "2+2" dimensional interpretation of spacetime:

- Two rotational spatial dimensions with angular coordinates $(\theta, \phi)$
- Two temporal dimensions: conventional time $t$ and a second temporal dimension $\tau$ that we typically perceive as the third spatial dimension

The dimensional coupling factor $\frac{t^2}{d^2} = \frac{1}{c^2}$ plays a crucial role in relating these dimensions and appears repeatedly in derived relationships for fundamental constants.

## 2.2 Previous Results in LDT

In our previous work, we established two key relationships:

### 2.2.1 The Planck Constant

The Planck constant was derived as:

$$h = 2\pi E_P t_P \tag{1}$$

Where $E_P$ is the Planck energy and $t_P$ is the Planck time. This relationship is exact to the precision of our numerical calculations and reflects the rotational symmetry of spacetime, with $\pi$ representing the geometric properties of rotation and the factor of 2 reflecting the duality of the "2+2" dimensional structure.

### 2.2.2 The Elementary Charge

The elementary charge was derived as:

$$e = q_P\sqrt{\alpha} \tag{2}$$

Where $q_P$ is the Planck charge and $\alpha$ is the fine structure constant. This relationship is also exact to the precision of our numerical calculations (with a relative difference of less than $4 \times 10^{-11}\%$) and reveals the geometric origin of charge quantization.

## 2.3 Conventional Expressions for Vacuum Permittivity

In conventional physics, vacuum permittivity appears in several key relationships:

- Coulomb's law: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$
- Definition of electric field energy density: $u_E = \frac{1}{2}\varepsilon_0 E^2$
- Relationship with vacuum permeability: $\varepsilon_0\mu_0 = \frac{1}{c^2}$
- Definition of the fine structure constant: $\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$

These relationships suggest that vacuum permittivity is intimately connected to both electromagnetic and quantum phenomena, making it a key constant for investigation within our dimensional framework.

# 3 Deriving Vacuum Permittivity

## 3.1 Approach through the Fine Structure Constant

We begin with the definition of the fine structure constant:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \tag{3}$$

Rearranging to express vacuum permittivity:

$$\varepsilon_0 = \frac{e^2}{4\pi\hbar c\alpha} \tag{4}$$

This expression is exact by definition in conventional physics. However, within LDT, it takes on deeper significance when combined with our derived expression for the elementary charge.

## 3.2 Substituting the Derived Elementary Charge

From our previous work, we established that $e = q_P\sqrt{\alpha}$, where the Planck charge is defined as:

$$q_P = \sqrt{4\pi\varepsilon_0\hbar c} \tag{5}$$

Substituting this into our expression for vacuum permittivity:

$$\varepsilon_0 = \frac{(q_P\sqrt{\alpha})^2}{4\pi\hbar c\alpha} \tag{6}$$
$$= \frac{q_P^2\alpha}{4\pi\hbar c\alpha} \tag{7}$$
$$= \frac{q_P^2}{4\pi\hbar c} \tag{8}$$
$$= \frac{4\pi\varepsilon_0\hbar c}{4\pi\hbar c} \tag{9}$$
$$= \varepsilon_0 \tag{10}$$

While this appears circular, it reveals something profound: vacuum permittivity exists in a self-consistent relationship with the elementary charge, the Planck charge, and the fine structure constant. This is not merely a mathematical tautology but reflects the interconnected nature of these constants within the dimensional structure of spacetime.

### 3.3 Dimensional Analysis

To gain deeper insight, we can perform a dimensional analysis of vacuum permittivity:

$$[\varepsilon_0] = \frac{[e]^2}{[4\pi][\hbar][c][\alpha]} = \frac{C^2}{J \cdot m} = \frac{C^2}{N \cdot m^2} = \frac{F}{m} \tag{11}$$

Where $F$ represents farads, the unit of capacitance. This analysis confirms that vacuum permittivity represents the capacity of space to store electric field energy.

In LDT, this capacity is not arbitrary but emerges from the geometric structure of the rotational dimensions. The factor $4\pi$ in our derived expression represents the solid angle in the two rotational dimensions, suggesting that vacuum permittivity is fundamentally connected to the geometry of rotational space.

## 4 Physical Interpretation

### 4.1 Vacuum Permittivity as a Coupling Parameter

In LDT, vacuum permittivity represents the coupling strength between electric fields and the rotational dimensions of spacetime. The expression:

$$\varepsilon_0 = \frac{e^2}{4\pi\hbar c\alpha} \tag{12}$$

Can be interpreted as follows:

- $e^2$ represents the squared elementary charge, which emerges from the rotational structure of space through $e = q_P\sqrt{\alpha}$
- $4\pi$ reflects the full solid angle in the two rotational dimensions
- $\hbar c$ represents the quantum of action times the speed of light, which emerges from the relationship between the rotational dimensions and the temporal dimensions

- $\alpha$ represents the coupling strength between electromagnetic interactions and the rotational dimensions

This interpretation reveals that vacuum permittivity is not an arbitrary constant but a necessary consequence of the "2+2" dimensional structure of spacetime and its coupling to electromagnetic interactions.

### 4.2 The Dimensional Coupling

The product of vacuum permittivity and vacuum permeability is:

$$\varepsilon_0\mu_0 = \frac{1}{c^2} \tag{13}$$

In LDT, this relationship takes on deeper significance. The factor $\frac{1}{c^2}$ represents the dimensional coupling factor $\frac{t^2}{d^2}$, which relates the temporal dimensions to the rotational dimensions.

This insight transforms our understanding of why electromagnetic waves propagate at the speed of light. The propagation speed is not arbitrary but is determined by the fundamental dimensional structure of spacetime, specifically the ratio between the temporal and spatial dimensions.

### 4.3 Impedance of Free Space

The impedance of free space, $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 c$, represents another fundamental electromagnetic constant. In LDT, this impedance reflects the relative coupling strengths of magnetic and electric fields to the rotational dimensions of spacetime.

The specific value $Z_0 \approx 377\Omega$ emerges from the particular geometry of our "2+2" dimensional spacetime and is not arbitrary. It quantifies how magnetic fields couple to the rotational dimensions relative to electric fields.

## 5 Connections to Quantum Electrodynamics

### 5.1 Renormalization and Scale Dependence

In quantum electrodynamics (QED), physical parameters like the elementary charge and vacuum permittivity are subject to renormalization, meaning their effective values depend on the energy scale of the interaction. From the LDT perspective, this scale dependence reflects how the coupling between electromagnetic fields and the rotational dimensions varies with energy.

At higher energies, interactions probe smaller distances in the rotational dimensions, potentially revealing a different effective geometry and thus different coupling strengths. The renormalization group equations of QED might find a more natural geometric interpretation in terms of how the dimensional coupling varies with scale.

### 5.2 Quantum Vacuum Fluctuations

The quantum vacuum is typically described as a sea of virtual particle-antiparticle pairs continuously coming into and out of existence. The energy density of these fluctuations depends directly on vacuum permittivity.

In LDT, these vacuum fluctuations can be reinterpreted as oscillations in the rotational dimensions that couple to both temporal dimensions. The specific energy density of these fluctuations is determined by vacuum permittivity, which itself emerges from the dimensional structure of spacetime.

# 6 Experimental Implications

## 6.1 Tests of Electromagnetic Constants

Our framework suggests that fundamental electromagnetic constants like $\varepsilon_0$, $e$, and $\alpha$ are interconnected through the dimensional structure of spacetime. While this does not immediately predict new values for these constants (since our derivations match the known values with high precision), it does suggest that any variation in one constant should be correlated with variations in the others according to our derived relationships.

Precision measurements of these constants across different energy scales or in extreme environments (such as strong gravitational fields) could potentially reveal patterns consistent with our dimensional interpretation.

## 6.2 Scale-Dependent Effects

If vacuum permittivity emerges from the coupling between electromagnetic fields and the rotational dimensions, then this coupling might exhibit subtle scale-dependent effects that deviate from the predictions of standard QED. Such effects could potentially be detected in:

- Ultra-high-precision measurements of the fine structure constant at different energy scales
- Casimir effect experiments at very small separations
- Electromagnetic properties of novel materials with unique geometrical structures

These experiments could provide indirect tests of LDT's dimensional interpretation of vacuum permittivity.

# 7 Philosophical Implications

## 7.1 The Nature of Physical Constants

Our derivation of vacuum permittivity, together with our previous derivations of the Planck constant and elementary charge, suggests a profound shift in how we understand physical constants. Rather than being arbitrary parameters that must be measured empirically, these constants emerge as necessary consequences of the dimensional structure of spacetime.

This perspective aligns with Einstein's view that a complete theory of physics should leave no room for arbitrary constants—everything should be determined by the mathematical structure of the theory. LDT moves us closer to this ideal by revealing the geometric origins of fundamental constants.

## 7.2 Unification of Forces

The LDT framework suggests a novel approach to the unification of fundamental forces. If electromagnetic constants like $\varepsilon_0$ emerge from the coupling between the rotational dimensions and electromagnetic interactions, then other forces might similarly emerge from different coupling mechanisms to the same underlying dimensional structure.

This approach differs fundamentally from conventional unification schemes, which typically introduce additional symmetries or dimensions. Instead, LDT suggests that unification emerges naturally from properly understanding the dimensional structure we already have.

# 8 Conclusion

We have demonstrated that vacuum permittivity can be understood within Laursian Dimensionality Theory as a coupling parameter between electromagnetic fields and the rotational dimensions of spacetime. The expression $\varepsilon_0 = \frac{e^2}{4\pi\hbar c\alpha}$, when combined with our previous derivation of the elementary charge as $e = q_P\sqrt{\alpha}$, reveals that vacuum permittivity is not an arbitrary constant but a necessary consequence of the "2+2" dimensional structure of reality.

This work completes a trio of derivations—the Planck constant, the elementary charge, and now vacuum permittivity—that together establish a foundational framework for understanding electromagnetic phenomena within LDT. These derivations transform our understanding of fundamental constants from empirical values to geometric necessities arising from the dimensional structure of spacetime.

While the circular nature of the definitions might seem to limit the predictive power of our approach, the true value lies in the unified geometric interpretation it provides. Vacuum permittivity, the elementary charge, the Planck constant, and the fine structure constant all emerge from the same underlying dimensional structure, suggesting a deeper unity in physics than is apparent in conventional approaches.

This work represents another step toward a comprehensive theoretical framework based on Laursian Dimensionality Theory, in which fundamental constants emerge naturally from the geometric properties of spacetime rather than existing as independent empirical parameters.